

New attractors and area codes

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ABSTRACT: In this note we give multiple examples of the recently proposed New Attractors describing supersymmetric flux vacua and non-supersymmetric extremal black holes in IIB string theory. Examples of non-supersymmetric extremal black hole attractors arise on a hypersurface in $WP_{1,1,1,1,2}^4$. For flux vacua on the orientifold of the same hypersurface existence of multiple basins of attraction is established. It is explained that certain fluxes may give rise to multiple supersymmetric flux vacua in a finite region on moduli space, say at the mirror of Landau-Ginzburg point and close to conifold point. This suggests the existence of multiple basins for flux vacua and domain walls in the landscape for a fixed flux and at interior points in moduli space.

KEYWORDS: Black Holes in String Theory, Superstring Vacua, Flux compactifications.

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1. Introduction

The supersymmetric extremal black hole attractors found in [1–4] (for a nice review see [5]) about ten years ago are still a subject of intensive research.

Another recent development, seemingly unrelated to black hole attractors, which attracts a lot of attention is flux compactification [6–8]. This development is reviewed nicely in [9] and [10] (see adequate bibliography here as well).

In this note we will address both black hole attractors and flux compactifications. Equations in both cases are pretty similar but the relation was unclear until recently, when the New Attractors were introduced in papers [11] and [12] (using [13]). These equations describe an analog of the supersymmetric extremal black hole attractor equations for supersymmetric flux vacua (and actually for non-supersymmetric ones too).

Here we choose a particular Calabi-Yau model (a hypersurface in $WP_{1,1,1,1,2}^4$) and show that the known examples of supersymmetric flux vacua (both Minkowski and AdS) of IIB string theory satisfy the New Attractor equations. Explicit examples of supersymmetric

flux vacua on Calabi-Yau three-folds were given in [14–17] (see [10] for complete bibliography). The particular classes of vacua checked here are at the mirror of Landau-Ginzburg point (to which we refer just as LG point later on) and in vicinity of the conifold point of the model, which are both in a finite region on the moduli space.

One observation of this note is the existence of area codes. This notion for black holes was introduced in [18]: for certain charges moduli space can be divided in "basins of attraction". Black hole horizon and black hole entropy become function of not only charges but also of basin on moduli space ("area"). Certain fluxes give rise to multiple vacua on moduli space and "area" of attraction may exist. Examples are given of such phenomena where, for fixed $F_{(3)}$ and $H_{(3)}$ flux, one supersymmetric flux vacuum is at the Landau-Ginzburg point while other is very close to the conifold. The values of the AdS cosmological constants in this case are close, but not the same. This suggests the existence of domain walls in the landscape (for further details see [19]).

Other observations we make have to do with the non-supersymmetric extremal black holes in IIB string theory introduced in [20]. Interesting recent developments on this subject appear in [21–23]. In [12] New Attractor equations for these non-supersymmetric black holes were proposed.

We give explicit examples of non-supersymmetric extremal black holes attractors and it is checked that the New Attractor equations for non-supersymmetric black holes work for them.

2. Review of new attractors for flux vacua

In papers [11, 12] the New Attractor equations for flux vacua were proposed. The following attractor equations for $N = 1$ supersymmetric flux vacua of IIB string theory were found:

$$\begin{pmatrix} h \\ f \end{pmatrix} = \begin{pmatrix} 2\text{Re}(Z\bar{\Pi}) \\ 2\text{Re}(Z\bar{\tau}\bar{\Pi}) \end{pmatrix} + \begin{pmatrix} 2\text{Re}(Z^{0I}\bar{D}_I\bar{\Pi}) \\ 2\text{Re}(\tau Z^{0I}\bar{D}_I\bar{\Pi}) \end{pmatrix} \quad (2.1)$$

where τ is the axio-dilaton; f and h are the magnetic and electric charges associated with the RR 3-form flux $F_{(3)}$ and NSNS 3-form flux $H_{(3)}$ of IIB respectively. Here Σ is the symplectic matrix $\Sigma \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Π is the covariantly holomorphic section of flat symplectic bundle of dimension $2h_{2,1}+2$ over the Kähler manifold which obeys the symplectic constraint

$$\Pi^\dagger \cdot \Sigma \cdot \Pi = i \quad (2.2)$$

The superpotential is defined to be

$$Z = (f - \tau h) \cdot \Sigma \cdot \Pi \quad (2.3)$$

while the mass matrix components are

$$Z^{0I} = D^0 D^I Z \quad (2.4)$$

with the flat derivatives are defined as $D^I = e_i^I D^i$ for the symplectic bundle and $D^0 = e_{\bar{\tau}}^0 D^{\bar{\tau}}$ for the axio-dilaton. Here we use orthonormal frame e_i^I and $e_{\bar{\tau}}^0$ such that $e_i^I g^{i\bar{j}} e_{\bar{j}}^{\bar{I}} = \delta^{I\bar{J}}$ and $e_{\bar{\tau}}^0 g^{\tau\bar{\tau}} e_{\bar{\tau}}^{\bar{0}} = \delta^{0\bar{0}}$ holds (where $g^{i\bar{j}}$ is the metric on complex structure moduli space and $g^{\tau\bar{\tau}}$ is the metric on axio-dilaton moduli space).

3. Review of supersymmetric flux vacua

As the properties of flux superpotentials on Calabi-Yau orientifolds in type IIB string theory have been reviewed many times, we will be brief. Our conventions are those of [15].

Consider a Calabi-Yau threefold M with $h_{2,1}$ complex structure deformations. Choose a symplectic basis $\{A^a, B_b\}$ for the $b_3 = 2h_{2,1} + 2$ three-cycles, $a, b = 1, \dots, h_{2,1} + 1$, with dual cohomology elements α_a, β^b such that:

$$\int_{A^a} \alpha_b = \delta_b^a, \quad \int_{B_b} \beta^a = -\delta_b^a, \quad \int_M \alpha_a \wedge \beta^b = \delta_a^b. \quad (3.1)$$

Fixing a normalization for the unique holomorphic three-form Ω , let us assemble the periods $z^a \equiv \int_{A^a} \Omega$, $\mathcal{G}_b \equiv \int_{B_b} \Omega$ into a b_3 -vector $\Pi(z) \equiv (\mathcal{G}_b, z^a)$. The z^a are taken as projective coordinates on the complex structure moduli space, with $\mathcal{G}_b = \partial_b \mathcal{G}(z)$. The Kähler potential \mathcal{K} for the z^a as well as the axio-dilaton $\tau \equiv C_0 + ie^{-\varphi}$ is

$$\mathcal{K} = -\log(i \int_M \Omega \wedge \bar{\Omega}) - \log(-i(\tau - \bar{\tau})) = -\log(-i\Pi^\dagger \cdot \Sigma \cdot \Pi) - \log(-i(\tau - \bar{\tau})), \quad (3.2)$$

where Σ is the symplectic matrix. The axio-dilaton and complex structure moduli take values in the moduli space \mathcal{M} ; a correct global description of the moduli space requires to identify points in \mathcal{M} related by modular symmetries.

Now consider nonzero fluxes of the RR and NSNS 3-form field strengths $F_{(3)}$ and $H_{(3)}$ over these three cycles, defining the integer-valued b_3 -vectors f and h via

$$F_{(3)} = -(2\pi)^2 \alpha' (f_a \alpha_a + f_{a+h_{2,1}+1} \beta^a), \quad H_{(3)} = -(2\pi)^2 \alpha' (h_a \alpha_a + h_{a+h_{2,1}+1} \beta^a). \quad (3.3)$$

These fluxes induce a superpotential for the complex structure moduli as well as the axio-dilaton:

$$W = \int_M G_{(3)} \wedge \Omega(z) = (2\pi)^2 \alpha' (f - \tau h) \cdot \Pi(z), \quad (3.4)$$

where $G_{(3)} \equiv F_{(3)} - \tau H_{(3)}$.

We will be interested exclusively in vacua satisfying the F-flatness conditions:

$$D_\tau W = D_a W = 0, \quad (3.5)$$

where $D_a W \equiv \partial_a W + W \partial_a \mathcal{K}$, and we have allowed a to run only over the $h_{2,1}$ inhomogeneous coordinates. This is alternatively

$$(f - \bar{\tau} h) \cdot \Pi(z) = (f - \tau h) \cdot (\partial_a \Pi + \Pi \partial_a \mathcal{K}) = 0. \quad (3.6)$$

These conditions force the complex structure to align such that the $(3, 0)$ and $(1, 2)$ parts of the fluxes vanish, leaving the fluxes “imaginary self-dual,” $*_6 G_{(3)} = iG_{(3)}$.

The fluxes also induce a contribution to the total D3-brane charge

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2} \int_M F_{(3)} \wedge H_{(3)} = f \cdot \Sigma \cdot h. \quad (3.7)$$

In the rest of the paper, we will set $(2\pi)^2\alpha' = 1$ for convenience. For vacua satisfying (3.5), the physical dilaton condition $\text{Im } \tau > 0$ implies that $N_{\text{flux}} > 0$. As the total charge on a compact manifold must vanish, sources of negative D3-charge must be present as well. For a given IIB orientifold compactification, a fixed amount of negative charge is induced by the orientifolds, leading to an effective bound on N_{flux} :

$$N_{\text{flux}} \leq L, \quad (3.8)$$

where, for instance in a IIB orientifold arising as a limit of a fourfold compactification of F-theory, L can be computed from the Euler character of the fourfold. Although the number of imaginary-self dual flux vacua is infinite, the set satisfying (3.8) for fixed L is in general finite.

In the absence of fluxes, a symmetry group $\mathcal{G} = SL(2, Z)_\tau \times \Gamma$ acts on the moduli space \mathcal{M} , where $SL(2, Z)_\tau$ is the S-duality of type IIB string theory and Γ is the modular group of the complex structure moduli space. Points on \mathcal{M} related by \mathcal{G} are considered equivalent, and a fundamental domain for the moduli space arises from dividing out by \mathcal{G} .

For the vacua we consider the fluxes are affected by \mathcal{G} as well. $SL(2, Z)_\tau$ acts in the ordinary way: given an $SL(2, Z)$ matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ we have

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} f \\ h \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} f \\ h \end{pmatrix}. \quad (3.9)$$

Under this transformation $(f - \tau h) \rightarrow (f - \tau h)/(c\tau + d)$, hence solutions of (3.6) are carried into other solutions, and N_{flux} (3.7) is preserved. The action of $SL(2, Z)$ generates a Kähler transformation on W (3.4) and \mathcal{K} (3.2):

$$W \rightarrow \Lambda W, \quad \mathcal{K} \rightarrow \mathcal{K} - \log \Lambda - \log \bar{\Lambda}, \quad (3.10)$$

with in this case $\Lambda = 1/(c\tau + d)$.

4. Supersymmetric flux vacua as new attractors

The relation of flux vacua notation in the previous section to the notation used in section §2 for New Attractors is as follows. The covariantly-holomorphic section is related to the holomorphic one as

$$\Pi \rightarrow e^{\frac{\mathcal{K}}{2}} \Pi(z) \quad (4.1)$$

The fluxes f and h of section §2 are related to fluxes of section §3 as follows

$$f \rightarrow \Sigma \cdot f \quad h \rightarrow \Sigma \cdot h \quad (4.2)$$

Finally the relation of the superpotential is

$$Z = e^{\frac{\mathcal{K}}{2}} W \quad (4.3)$$

New Attractors (2.1) can now be rewritten in terms of flux vacua notations as

$$\begin{pmatrix} \Sigma \cdot h \\ \Sigma \cdot f \end{pmatrix} = e^{\mathcal{K}} \begin{pmatrix} 2\text{Re}(W\bar{\Pi}) \\ 2\text{Re}(\bar{\tau}W\bar{\Pi}) \end{pmatrix} \Big|_{DW=0} + e^{\mathcal{K}} \begin{pmatrix} 2\text{Re}(e_{\underline{0}}^{\tau} e_1^{\psi} e_{\underline{1}}^{\bar{\psi}} D_{\psi} D_{\tau} W \overline{D_{\psi} \bar{\Pi}}) \\ 2\text{Re}(e_{\underline{0}}^{\tau} e_1^{\psi} e_{\underline{1}}^{\bar{\psi}} \tau D_{\psi} D_{\tau} W \overline{D_{\psi} \bar{\Pi}}) \end{pmatrix} \Big|_{DW=0} \quad (4.4)$$

where for simplicity we assume that we have only one complex structure modulus ψ (as will be the case in examples in the remaining part of this note) and

$$e_1^{\psi} e_{\underline{1}}^{\bar{\psi}} = g^{\psi\bar{\psi}} = \frac{1}{\partial_{\psi} \partial_{\bar{\psi}} \mathcal{K}} \quad e_{\underline{0}}^{\tau} = -(\tau - \bar{\tau}). \quad (4.5)$$

5. Flux vacua as new attractors in a simple Calabi-Yau hypersurface

Consider a Calabi-Yau threefold defined as a hypersurface in a weighted projective space. The Calabi-Yau threefold of interest is defined by the equation

$$\sum_{i=1}^4 x_i^6 + 2x_0^3 - 6\psi x_0 x_1 x_2 x_3 x_4 = 0 \quad x_i \in WP_{1,1,1,1,2}^4. \quad (5.1)$$

On its moduli space it has Landau-Ginzburg, conifold and large complex structure points. We will analyze the flux vacua at the Landau-Ginzburg point and in vicinity of the conifold point. Both of these points are at a finite distance in moduli space.

Moduli space and periods of one parametric models were first studied in [24–26]. We will use their results.

5.1 Flux vacua at Landau-Ginzburg point

In general the Landau-Ginzburg point is a very special point in the moduli space, where the number of vacua with $W = 0$ and with discrete symmetries can be of the same order as the total number of vacua, when nonzero [16].

Near the Landau-Ginzburg point $\psi = 0$ the periods admit expansion in a Picard-Fuchs basis

$$w_i(\psi) = \frac{(2\pi i)^3}{6} \sum_{n=1}^{\infty} \frac{\exp(\frac{5\pi i}{6} n) \Gamma(\frac{n}{6})}{\Gamma(n) \Gamma(1 - \frac{n}{6})^3 \Gamma(1 - \frac{n}{3})} \left(\frac{6\alpha^i}{2^{1/3}} \right)^n \psi^{n-1}. \quad (5.2)$$

This is valid for $|\psi| < 1$, where α is the 6th root of unity

$$\alpha = \exp\left(\frac{2\pi i}{6}\right). \quad (5.3)$$

In symplectic basis the periods then have the expansion

$$\Pi = \begin{pmatrix} \mathcal{G}_1 \\ \mathcal{G}_2 \\ z^1 \\ z^2 \end{pmatrix} = m \cdot \begin{pmatrix} w_2 \\ w_1 \\ w_0 \\ w_5 \end{pmatrix} = c_0 p_0 + c_1 \psi p_1 + O(|\psi|^2) \quad (5.4)$$

around the LG point ψ . Here c_0, c_1 are constants and the matrix of transformation from the Picard-Fuchs to the symplectic basis is given by

$$m = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \quad (5.5)$$

and the following definitions are introduced

$$p_0 = \begin{pmatrix} \alpha^2 \\ \alpha \\ 1 \\ \alpha^5 \end{pmatrix} \quad p_1 = \begin{pmatrix} \alpha^4 \\ \alpha^2 \\ 1 \\ \alpha^4 \end{pmatrix}. \quad (5.6)$$

The monodromy group, Γ , of the complex structure moduli space has two generators: A , which generates phase rotations $\psi \rightarrow \alpha\psi$ with $\alpha = \exp(2\pi i/6)$ around the LG point at $\psi = 0$, and T which corresponds to the logarithmic monodromy $\mathcal{G}_2 \rightarrow \mathcal{G}_2 + z^2$ around the conifold singularity $\psi = 1$. By itself, A generates a $\mathbf{Z}_6 \subset \Gamma$ subgroup, with an associated fixed point at $\psi = 0$; T , on the other hand, is of infinite order.

F-flatness condition at $\psi = 0$ reduces to

$$D_\tau W = (f - \bar{\tau}h) \cdot p_0 = 0 \quad (5.7)$$

$$D_\psi W = (f - \tau h) \cdot p_1 = 0. \quad (5.8)$$

The monodromy matrix A generates rotations by a root of unity around $\psi = 0$:

$$A\Pi(\psi) = \alpha\Pi(\alpha\psi), \quad (5.9)$$

and is explicitly given by

$$A = \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ -3 & -3 & 1 & 3 \\ -6 & 4 & 1 & -3 \end{pmatrix}. \quad (5.10)$$

Vacua exist at the LG point as long as

$$\tau = t_1 + \alpha t_2 \quad (5.11)$$

where t_1 and t_2 are rational [16].

The LG point is a fixed point for $\mathbf{Z}_6 \subset \Gamma$, so one may hope that this symmetry is preserved in the low-energy theory. Additionally, the \mathbf{Z}_2 and \mathbf{Z}_3 points on the dilaton moduli space are also potential sources of low-energy symmetry. Only \mathbf{Z}_3 is accessible though.

5.2 (0, 3) flux vacua

In addition to F-flatness these flux vacua satisfy the condition

$$D_\tau D_\psi W = 0 \quad (5.12)$$

which results in the following constraint for fluxes which define the vacuum

$$f \cdot p_1 = h \cdot p_1 = 0 \quad (5.13)$$

This is solved by the following

$$h = -h \cdot A^3 \quad \rightarrow \quad h = (-3h_3 + 3h_4, h_3, h_3, h_4) \quad (5.14)$$

$$f = -f \cdot A^3 \quad \rightarrow \quad f = (-3f_3 + 3f_4, f_3, f_3, f_4). \quad (5.15)$$

Hence in this case we have $\mathbf{Z}_2 \subset \mathbf{Z}_6$ preserved as a true symmetry (see [16] for details):

$$W(\tau, -\psi) = W(\tau, \psi). \quad (5.16)$$

Let us now see if the New Attractor equations hold for these flux vacua. The second term in New Attractors (4.4) vanishes and they become

$$\begin{pmatrix} \Sigma \cdot h \\ \Sigma \cdot f \end{pmatrix} = e^{\mathcal{K}} \begin{pmatrix} 2\text{Re}(W\bar{\Pi}) \\ 2\text{Re}(\bar{\tau}W\bar{\Pi}) \end{pmatrix} \Bigg|_{\substack{\psi = 0 \\ \tau = \frac{f \cdot p_0}{h \cdot p_0}}} \quad (5.17)$$

We checked this using computer algebra as we do most of the checks in this note.

5.3 (2, 1) flux vacua

In addition to F-flatness these vacua satisfy the condition $W = 0$ which results in the following necessary and sufficient condition

$$f \cdot p_0 = h \cdot p_0 = 0. \quad (5.18)$$

This is solved by the following choice of fluxes

$$h = h \cdot A^3 \quad \rightarrow \quad h = (-3h_3 + h_4, 3h_3, h_3, h_4). \quad (5.19)$$

$$f = f \cdot A^3 \quad \rightarrow \quad f = (-3f_3 + f_4, 3f_3, f_3, f_4) \quad (5.20)$$

Hence these vacua have a \mathbf{Z}_2 R-symmetry (see [16] for details)

$$W(\tau, -\psi) = -W(\tau, \psi) \quad (5.21)$$

and it is easy to see that this ensures $W(\tau, \psi = 0) = 0$. This is the R-symmetry “responsible” for the vanishing of the vacuum superpotential.

In checking the New Attractor equations we see that the first term now vanishes and they become

$$\begin{pmatrix} \Sigma \cdot h \\ \Sigma \cdot f \end{pmatrix} = e^{\mathcal{K}} \begin{pmatrix} 2\text{Re}(g^{\psi\bar{\psi}} e_{\underline{0}}^\tau D_\psi D_\tau W \overline{D_\psi \bar{\Pi}}) \\ 2\text{Re}(g^{\psi\bar{\psi}} e_{\underline{0}}^\tau \tau D_\psi D_\tau W \overline{D_\psi \bar{\Pi}}) \end{pmatrix} \Bigg|_{\tau = \frac{f \cdot p_1}{h \cdot p_1}} \quad (5.22)$$

where the metric at Landau-Ginzburg point on complex structure moduli space is given by

$$g^{\psi\bar{\psi}} = \frac{|c_1|^2}{3|c_0|^2}. \quad (5.23)$$

Using computer algebra it is easy to check that these equations are indeed satisfied.

5.4 (0, 3) + (2, 1) flux vacua

Generic flux vacua (with W and $D_\tau D_\psi W$ not equal zero generically) are defined by the condition

$$f = t_1 h - t_2 h \cdot A^2 \tag{5.24}$$

for any integral h and rational t_1, t_2 chosen so that f is integral and the axio-dilaton is given by

$$\tau = t_1 + \alpha t_2. \tag{5.25}$$

In this case the New Attractors (4.4) should work as well, once again using computer algebra, it is easy to check that they are satisfied.

5.5 Flux vacua at conifold region

Let us now study supersymmetric flux vacua in conifold region of the $WP_{1,1,1,1,2}^4$ model.

In a symplectic basis the periods in the vicinity of the conifold point $\psi = 1$ can be given to first order by the following expressions (here $x \equiv 1 - \psi$ and $|x| \ll 1$)

$$\begin{aligned} \mathcal{G}_1(x) &= (2\pi i)^3 [a_0 + a_1 x + O(x^2)], \\ \mathcal{G}_2(x) &= \frac{z^2(x)}{2\pi i} \ln(x) + (2\pi i)^3 [b_0 + b_1 x + O(x^2)], \\ z^1(x) &= (2\pi i)^3 [c_0 + c_1 x + O(x^2)], \\ z^2(x) &= (2\pi i)^3 [d_0 + d_1 x + O(x^2)]. \end{aligned} \tag{5.26}$$

Where the constants can be approximated by the following numbers

$$\begin{aligned} a_0 &= 1.501i, & c_0 &= -5.087 + 6.754i, \\ a_1 &= -0.914i, & c_1 &= 4.261 - 4.112i, \\ b_0 &= 1.056, & d_0 &= 0, \\ b_1 &= -0.344 - 0.827i, & d_1 &= -1.654i. \end{aligned} \tag{5.27}$$

The Kähler potential for the complex structure modulus is given by

$$K_\psi = -\ln[\mu_0 + \mu_1 x + \bar{\mu}_1 \bar{x} + \mu_2 |x|^2 \ln |x|^2 + \mu_3 |x|^2 + \mu_4 x^2 + \bar{\mu}_4 \bar{x}^2 + O(|x|^3 \ln |x|)], \tag{5.28}$$

with the relevant constants μ_0, μ_1, μ_2 and μ_3 given by

$$\begin{aligned} \mu_0 &= i(2\pi)^6 (a_0 \bar{c}_0 - c_0 \bar{a}_0), & \mu_1 &= i(2\pi)^6 (\bar{c}_0 a_1 - c_1 \bar{a}_0 - d_1 \bar{b}_0), \\ \mu_2 &= (2\pi)^5 |d_1|^2, & \mu_3 &= i(2\pi)^6 (\bar{c}_1 a_1 - \bar{a}_1 c_1 + \bar{d}_1 b_1 - \bar{b}_1 d_1). \end{aligned} \tag{5.29}$$

One finds the following expression for the Kähler metric

$$g_{x\bar{x}} = -\frac{\mu_2}{\mu_0} \ln |x|^2 + \left(\frac{|\mu_1|^2}{\mu_0^2} - \frac{2\mu_2 + \mu_3}{\mu_0} \right) + O(|x| \ln |x|). \tag{5.30}$$

In computing Kähler covariantized derivatives with respect to ψ it is also useful to note that

$$\partial_x K_\psi = -\frac{\mu_1}{\mu_0} - \frac{\mu_2}{\mu_0} \bar{x} \ln |x|^2 + O(x). \tag{5.31}$$

5.6 Flux vacua as new attractors at the conifold point

The approximate equations for supersymmetric flux vacua that are very close to conifold point take the form [15]

$$D_\tau W = 0 \quad \Rightarrow \quad \tau = \frac{f \cdot \Pi^\dagger}{h \cdot \Pi^\dagger} = \frac{f_1 \bar{a}_0 + f_2 \bar{b}_0 + f_3 \bar{c}_0}{h_1 \bar{a}_0 + h_2 \bar{b}_0 + h_3 \bar{c}_0} + O(|x| \ln |x|); \quad (5.32)$$

$$D_\psi W = 0 \quad \Rightarrow \quad \ln(x) = -\frac{2\pi i}{d_1} \left[\frac{(f_1 - \tau h_1)(a_1 - \frac{\mu_1}{\mu_0} a_0) + (f_2 - \tau h_2)(b_1 - \frac{\mu_1}{\mu_0} b_0)}{f_2 - \tau h_2} + \frac{(f_3 - \tau h_3)(c_1 - \frac{\mu_1}{\mu_0} c_0) + (f_4 - \tau h_4)d_1}{f_2 - \tau h_2} \right] - 1. \quad (5.33)$$

One can make Monte Carlo simulations of such vacua. This was done in [15] where the attractive nature of conifold point was established. We will take one particular choice of fluxes to illustrate that the New Attractors (4.4) work. For the choice of fluxes

$$f = \{61, 10, -11, -15\} \quad h = \{3, 4, 4, 11\} \quad (5.34)$$

and using (5.32), (5.33), one finds a supersymmetric vacuum deep in the conifold region with

$$\ln(1 - \psi) = -7.29 - 0.71i \quad \tau = -0.42 + 1.90i \quad (5.35)$$

One may check that the F-flatness conditions

$$D_\tau W \approx 0 \quad D_\psi W \approx 0 \quad (5.36)$$

hold with precision $O(10^{-2})$ and New Attractors hold with precision $O(10^{-3})$.

6. Area codes for flux vacua

A new phenomena that we observe in this note is that certain fluxes may give rise to multiple minima within a finite distance on the moduli space. This suggests the existence of area codes and basins of attraction for these flux vacua.

We give an illustration of these multiple basin attractors here. For simplicity let us consider flux vacua with the \mathbf{Z}_3 symmetry on the dilaton moduli space being preserved in the low-energy action. In order for fluxes to permit $\tau = \alpha$ as a vacuum we must restrict the Landau-Ginzburg solutions [16] to fluxes satisfying

$$f = -h \cdot A^2. \quad (6.1)$$

To be specific let us take the $H_{(3)}$ flux to be

$$h = \{-72, -3, 13, -4\} \quad (6.2)$$

Note this supersymmetric flux vacuum with enhanced symmetry satisfies the New Attractors (4.4) of course.

It turns out, using (5.32) and (5.33) , that this flux give rise to a vacuum deep in conifold region with

$$\tau = 0.37 + 1.11i \quad \ln(1 - \psi) = -18.47 - 0.37i \quad (6.3)$$

Computer algebra confirms that the F-flatness conditions and the New Attractor equations hold with precision $O(10^{-3})$.

This means that for flux choice (6.2) there are at least two supersymmetric AdS flux vacua, with cosmological constant $V = -3e^{\mathcal{K}}|W|^2$ given by

$$V_{\text{LG}} = -2080.5 \quad V_{\text{conifold}} \simeq -2281.7 \quad (6.4)$$

which are not the same but are close.

This suggests the possible existence of basins of attraction and domain walls in the landscape domain walls between two flux vacua which arise for a fixed flux and both of which are at interior points in moduli space.

7. Non-supersymmetric extremal black holes

In this section we switch gears and consider the non-supersymmetric extremal black holes proposed in [20] and rediscovered and developed more recently in [21–23].

Let us consider the same $WP_{1,1,1,1,2}^4$ model and construct non-supersymmetric extremal black hole attractors at the Landau-Ginzburg point of this model.

In $\mathcal{N} = 2$ supersymmetric theory the effective black hole potential can be expressed as [20, 21]

$$V_{\text{BH}} = e^{\mathcal{K}}(|DW|^2 + |W|^2) \quad (7.1)$$

in terms of a Kähler potential given by

$$\mathcal{K} = -\log(-i\Pi^\dagger \cdot \Sigma \cdot \Pi) \quad (7.2)$$

and a superpotential which reads as

$$W = \int_M F_{(3)} \wedge \Omega = f \cdot \Pi \quad (7.3)$$

where f is the $F_{(3)}$ magnetic and electric flux.

Non-supersymmetric black hole attractors are just critical points of effective black hole potential [20]

$$\partial_i V_{\text{BH}} = 0 \quad (7.4)$$

and subject to attractors [21] if

$$M_{ij} = \frac{1}{2}\partial_i\partial_j V_{\text{BH}} > 0. \quad (7.5)$$

In this case the resulting black hall entropy is just given by

$$S_{\text{BH}} = \pi V_{\text{BH}} \quad (7.6)$$

at the minimum with a positive definite mass matrix.

Let us now go back to our model with one complex structure modulus. It turns out that this minimum of the effective black hole potential

$$V_{\text{eff}}(\psi) = e^{\mathcal{K}}(G^{\psi\bar{\psi}}|D_{\psi}W| + |W|^2) \quad \partial_{\psi}V_{\text{eff}}(\psi) = 0 \quad (7.7)$$

happens to be at Landau-Ginzburg point for fluxes

$$f = (-3f_3 + f_4, 3f_3, f_3, f_4). \quad (7.8)$$

This is equivalent to $f = f \cdot A^3$ in terms of Landau-Ginzburg monodromy matrix A defined in (5.10).

The mass matrix for these fluxes is found to be

$$\partial_{\psi}\partial_{\psi}V_{\text{BH}}(\psi)|_{\psi=0} = 0 \quad \partial_{\psi}\partial_{\bar{\psi}}V_{\text{BH}}(\psi)|_{\psi=0} = 2G_{\psi\bar{\psi}}|_{\psi=0}V_{\text{BH}}|_{\psi=0} \quad (7.9)$$

where the effective black hole potential at the attractor point is

$$V_{\text{BH}}(0) = \frac{2}{\sqrt{3}}(3f_3^2 + 3f_3f_4 + f_4^2). \quad (7.10)$$

This means that these critical points are indeed attractors for all possible fluxes (7.8). Note also that for this case we have $W = 0$, though this is non-generic as we see from the examples in [23].

7.1 New attractors for non-supersymmetric extremal black holes

Let us finally check that the recently proposed New Attractor equations for non-supersymmetric extremal black holes [12] holds. This equation is

$$f = 2e^{\mathcal{K}}\text{Im}(W\bar{\Pi} - G^{a\bar{a}}D_aW\bar{D}_{\bar{a}}\bar{\Pi}). \quad (7.11)$$

In our simple case of a one dimensional moduli space (and our particular choice of fluxes) these equations become

$$\Sigma \cdot f = 2e^{\mathcal{K}}\text{Im}(W\bar{\Pi} - G^{\psi\bar{\psi}}D_{\psi}W\bar{D}_{\bar{\psi}}\bar{\Pi}) \quad (7.12)$$

It is a matter of simple algebra to check that these equations are indeed satisfied for the solutions above.

We finish our note with the remark that through the explicit examples in the sections above we have illustrated phenomena which certainly deserve further attention.

8. Conclusion

In this note we have found multiple examples of supersymmetric flux vacua which satisfy new supersymmetric attractor equations. We have also found examples of non-supersymmetric black hole attractors which solve the corresponding non-supersymmetric New Attractor equations. In conclusion we want to make one observation concerning the flux vacua.

Though the existence of domain walls of various sorts in the landscape was already known; e.g. the domain wall between the KKLT vacuum and infinity, or the domain walls which jump the quantized RR and NS flux and are given by D and NS branes wrapping 3-cycles. In this note however we are giving the first signs of existence of domain wall between two flux vacua which arise for a fixed flux and both of which are at interior points in moduli space.

With this regard we should say that a first bottom-up pass at the "measure problem" for vacuum selection would clearly involve doing statistics of the sizes of basins of attraction (in the metric on CY moduli space) for the various vacua which arise at a fixed flux.

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